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# EXPERIMENTAL INVESTIGATION OF THE DECOMPOSITION OF A CYLINDRICAL LAYER OF A MAGNETIZING LIQUID UNDER THE ACTION OF MAGNETIC FORCES

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UDC 532.595:538.4

Questions of the stability of jet flows of liquid, in connection with their various technical applications, have attracted the attention of investigators in the field of the hydrodynamics of continuous media [1-9]. Magnetizing liquids hold out the possibility of effective action at an interface, using a magnetic field [6-9]. It has been established in theoretical work [10-12] that a homogeneous magnetic field tangential to a surface stabilizes jet and film-type flows, while at the same time, a field directed along a normal has a destabilizing action. For example, the field of the conductor with a current is always tangential to the surface of a cylindrical layer and, consequently, exerts a stabilizing effect on the surface of a magnetizing liquid. In this case, there arises the possibility of modeling a jet and studying its characteristics in a static statement [13]. The present investigation is a continuation of [9].

In a linear approximation, the process of the decomposition of a cylindrical layer of a magnetizing liquid can be described by a dispersion equation for infinitely small perturbations, under the assumption that the radius of the conductor is small [12]:

$$\omega^2 = I_1 \alpha k(\mathrm{Bo}_m - 1 + k^2)/I_0 \rho a^3,$$

where $\rho$ is the density of the magnetizing liquid; $I_0$ and $I_1$ are Bessel functions of an imaginary argument; k is the wave number made dimensionless with respect to the radius of the column of liquid; $\alpha$ is the coefficient of surface tension at the interface of the magnetizing liquid with the surrounding medium; $a$ is the radius of the cylinder of magnetizing liquid.

The dimensionless parameters $\mathrm{Bo}_m = \mu_0 MGa^3/\alpha$, in which $\mu_0$ is the magnetic permeability of a vacuum; M is the magnetizability of the liquid; G is the gradient of the intensity of the magnetic field, and analogous to the well-known Bond number $\mathrm{Bo} = \rho g a^2/\alpha$; in [13] it was called the magnetic Bond number. This number is the ratio of the pressure induced by the volumetric magnetic force $\mu_0 MG$ to the pressure set up at the interface by the forces of surface tension. With $\mathrm{Bo}_m > 1$, the layer of perturbations is stable with any given arbitrary perturbations; if the condition $\mathrm{Bo}_m < 1$ is satisfied, the cylindrical layer falls apart into individual drops. For a conductor with a current $G = \mathcal{J}/2\pi a^2$ ($\mathcal{J}$ is the current through the conductor). Setting $M = \chi H$, which is valid with small values of the intensity of the magnetic field H, and writing H in the form $H = \mathcal{J}/2\pi a$, we obtain

$$\mathrm{Bo}_m = \mu_0 \chi \mathcal{J}^2/4\pi^2 a,$$

where $\chi$ is the magnetic susceptibility of the magnetizing liquid.

To describe the effect of an external homogeneous magnetic field on the stability of the cylindrical layer, we introduce the dimensionless complex $S = \mu_0 M^2 a/\alpha$, used in working up the results of an investigation of a suspended drop in a homogeneous magnetic field [14].

Experiments on the investigation of the stability of a cylindrical layer and of a study of the effect of an external homogeneous magnetic field perpendicular to the axis of the cylinder, on it, were made in a glass vessel with horizontal dimensions of 240 × 40 mm and a height of 40 mm. The action of an external magnetic force tangential to the surface of the cylinder was investigated in a vessel with horizontal dimensions of 40 × 40 mm and a height of 130 mm. Along the long axis of the vessels there was installed a horizontal hollow conductor with an external radius of 1 mm, made out of a nonmagnetic material and cooled by flow-through water with a constant temperature. The length of the cylindrical layer of magnetizing liquid in a horizontal vessel was 160 mm, and in a vertical vessel 80 mm.

The working liquid was a ferroliquid, whose saturation magnetization was 27 kA/m, density $\rho = 1.25 \cdot 10^3$ kg/m³, and magnetic susceptability $\chi = 1.242$. The gravitational forces at the surface of the cylindrical layer was compensated by

Minsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 98-105, May-June, 1980. Original article submitted May 10, 1979.

0021-8944/80/2103-0371$07.50  

filling the vessels with an aqueous solution of glycerin, with a density equal to the density of the ferroliquid. The transparency of the aqueous solution of glycerin permitted visual observations and cine photography. The coefficient of surface tension at the interface of these two media, measured by the method of the breakaway of a drop, was $\alpha = 13 \cdot 10^{-3}$ N/m.

With a study of the effect of an external homogeneous magnetic field on the form of a cylindrical layer of a magnetizing liquid, the vessels were put between the poles of an electromagnet whose diameter was 450 mm, making it possible to obtain a vertically directed homogeneous magnetic field, whose intensity could be varied in a range of 0-287 kA/m. The inhomogeneity of the magnetic field did not exceed 2.4% in a region measuring 200 × 200 mm.

With investigation of the stability of the cylindrical layer, a direct current was passed through the conductor, setting up an axially symmetrical magnetic field with a radial gradient of the intensity. A value of the current was established corresponding to supercritical values of the magnetic Bond number $Bo_m > 1$. In this case, the ferroliquid assumes the stable form of a cylinder around the conductor. Then, by the connection of shunts, the current in the conductor was sharply decreased to values with which the Bond number was less than the critical ($Bo_m < 1$), and the layer went over into a stable state. The radius of the layer was measured using a KM-8 kathetometer. The process of the development of instability was recorded on cine film, which was then processed in an MIR-12 measuring microscope.

Analysis of the experimental data made it possible to obtain quantitative information of the process of the decomposition of a cylindrical layer of a ferroliquid, that is: the wavelength of the most dangerous perturbations, the time of their development, as well as to bring out the special characteristics of this process, e.g., the coalescence of drops with different values of the determining parameters, the formation of satellites, etc.

The radius of the cylindrical layer varied from 1.2 to 5 mm, and the range of change in the magnetic Bond number was within the limits of 0-3.

As was shown in [13], the surface of the cylindrical layer, with a jumpwise change in the feed current, becomes wavy with the formation of protrusions and depressions (drops connected by necks). The wavelength $\lambda$, which represents the distance between adjacent drops, referred to the perimeter of the layer, increases with a rise in the $Bo_m$ number. Figure 1 shows different situations arising with the decomposition of the cylindrical layer. The $Bo_m$ number decreases from the top downward, from values exceeding unity to zero. Under these circumstances, the wavelength decreases from 4.1 to 1.5. It must be noted that the attainment of a given value of $\lambda$ is possible in a definite range of $Bo_m$ numbers, which is connected with a finite length of the layer of magnetizing liquid. At the boundaries between adjacent ranges, with a change in $Bo_m$, $\lambda$ changes jumpwise. With $Bo_m$ numbers close to zero, the drops into which the layer decomposes, have a form close to spherical, with only small deviations from sphericity along their horizontal axis. The elongation of the drops along the horizontal increases with a rise in the $Bo_m$ number, and, with $Bo_m$ numbers close to unity, they take on the form of an ellipsoid.

Figure 2 gives typical curves of the development of a drop and a neck with time. Along the axis of ordinates there is plotted the value of the deviation A from the initial position of the cylindrical surface of the magnetizing liquid. The continuous lines connect points corresponding to the drop, the broken lines connect open points characterizing the development of a neck. Curves 1, 2 correspond to a Bond number $Bo_m = 0.127$; curves 5, 6 to a Bond number $Bo_m = 0.586$. Curves 3, 4 coincide, here the Bond number $Bo_m = 0.25$. The measurements were made for a central drop with respect to the layer and to the neck between it and the adjacent drop. With small $Bo_m$ numbers, the development of the instability takes place considerably more rapidly than with $Bo_m$ close to unity, which is connected with a lower value, inhibiting the development of a magnetic force. With an increase in $Bo_m$, the diameter of the drops decreases, and the diameter of the necks increases. In the initial selection of the development of instability, there is recorded an exponential rise in the perturbations, which is in agreement with the linear theory [1, 2]. The time of the complete development of the instability, which was determined from the stopping of the growth of the drops and the necks, was from 3.5 sec with $Bo_m = 0$ to 20 sec with $Bo_m = 0.653$.

With $Bo_m \geqslant 0.25$, the drops grow more rapidly than the necks, and, when this value is exceeded, the drops develop slower than the necks. For a jet of nonmagnetic liquid, it has been established theoretically and confirmed experimentally (see, e.g., [2, 3]) that the necks always develop faster than the drops. This contradiction is connected with the existence of a solid boundary inside the cylindrical layer. With the decomposition of the jet, in the process of development of the necks, the capillary forces increase and, by the same token, accelerate the process. However, the presence of a solid wall inside the layer prevents this, and the development time of the necks rises. In addition, with a decrease in the diameter of the necks, there is a sharp increase in the volumetric magnetic forces, which also prevents their development, since their value is inversely proportional to the distance from the axis to the third power. With large Bond numbers, the principal role in the process of the decomposition of the layer starts to be played by volumetric forces and, obviously as a result of their inhomogeneous distribution over the radius, the necks develop more rapidly.

The process of the decomposition of the cylindrical layer with time is graphically shown on the cine photos given in Fig. 3. On the cine photo in Fig. 3a, the time between adjacent frames is 1.4 sec. The development of instability starts near the vertical walls limiting the length of the layer, which is explained by the effect of the edge wetting angle. With the passage of time, the perturbations are propagated over the whole length of the layer. The total time of decomposition of the cylindrical layer attains 8.5 sec (Fig. 3a).

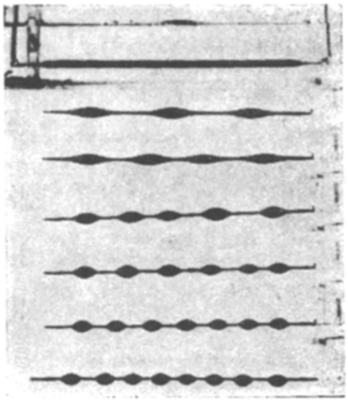

Fig. 1

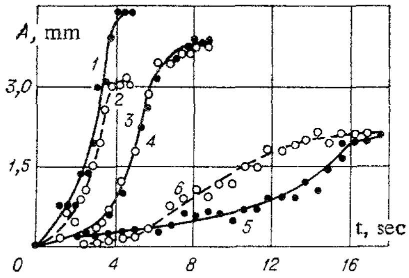


Fig. 2

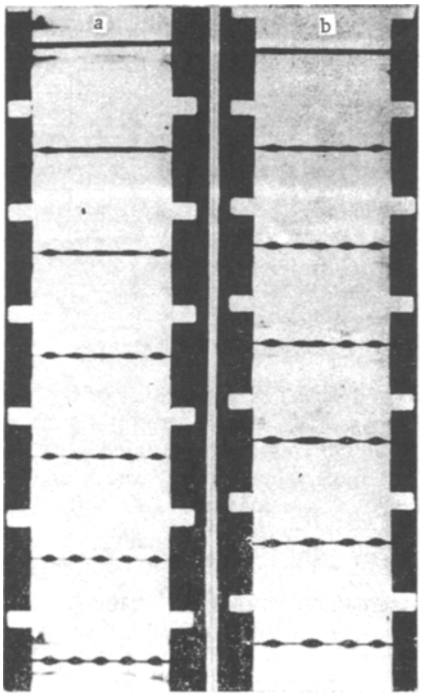


Fig. 3

With $Bo_m$ numbers close to the boundaries of the range of existence of instability with a given wavelength, there is the possibility of the coalescence of two neighboring drops. In the first stage of this process (Fig. 3b), there is the usual development of the drops and necks up to their normal dimensions for the given value of $Bo_m$. In the second stage, two adjacent drops start to coalesce; the diameter of the neck between them increases until a new drop is formed as a result of the coalescence of the two adjacent drops. The diameter of the new drop and the time of its development are almost 1.5 times as great as the diameter of ordinary drops and the time of their development for a given value of $Bo_m$. The time interval between adjacent frames of the cine photo of Fig. 3b is 4.1 sec.

With small $Bo_m$ numbers, there is the possibility of the appearance of secondary drops, so-called satellites which arise between the main drops. The satellites appear in the final stage of the development of instability, and are formed in the following way. At first, the diameter of the neck decreases analogously to what is shown in Fig. 2. Then, at half the distance between neighboring drops, the decrease in the neck stops, and, at the edges, there appear two new necks; here, their diameter starts to decrease rapidly. The necks forming again stop developing simultaneously with the cessation of the growth of the main drops. The diameter of the satellites is, as a rule, 0.1-0.4 of the diameter of the main drops.

When a stable cylindrical layer of a magnetizing liquid ($Bo_m > 1$) is put into an external homogeneous magnetic field, perpendicular to its axis, starting from a determined value of the intensity of this field $H_*$ and, correspondingly, of

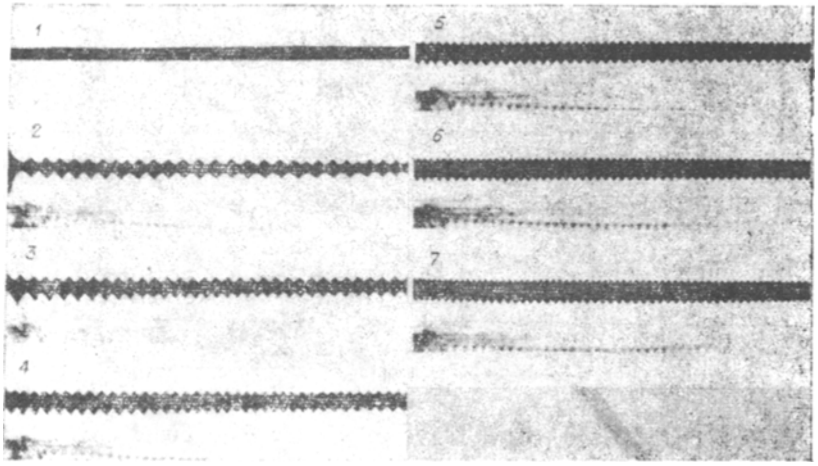


Fig. 4

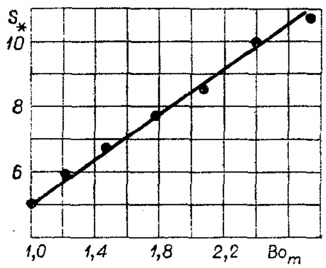


Fig. 5

S*, instability of the surface develops, due to this field. With values of S < S*, with a rise in S, the cylindrical layer extends along the lines of force of the field as a whole. In this case, in a cross section perpendicular to the axis, the layer takes on the form of an ellipse. This stage of the development is analogous to the elongation of a suspended drop along a field [14]. Then, with the attainment of a critical value S*, the surface of the elliptical cylinder becomes unstable, and cone-shaped peaks appear on it, similar to the peaks appearing as a result of the instability of a flat surface [6, 7, 15].

Figure 4 demonstrates the change in the form of an originally cylindrical layer with an increase in the intensity of the magnetic field. In the upper photograph, the intensity $H_1 = 0$, and, in the following ones, decreases from $H_2 = 10.2$ kA/m to $H_7 = 103$ kA/m. The wavelength of the perturbations of the surface decreases with an increase in H. The difference in the wavelengths of the perturbations at the upper and lower parts of the cylinder (Fig. 4) is due to the difference in the densities of the liquids. If the density of the magnetizing liquid is greater than the density of the aqueous solution of glycerin, then, the wavelength of the perturbations at the upper part of the surface of the cylinder is less than at the lower, and the contrary.

With a sufficiently great value of H (~100 kA/m) at the tops of the cone-shaped peaks formed there appear secondary small peaks in the form of needles, which disappear with a decrease in the intensity of the external magnetic field.

The critical value of the parameter S*, with which surface instability develops, depends on the value of the magnetic Bond number $Bo_m$, i.e., an increase in the inhibiting volumetric magnetic force, due to the axially symmetrical magnetic field with a current, the intensity of the magnetic field required for the development of surface instability rises. In the region lying below the curve $S_* = S_*(Bo_m)$, the cylindrical layer of liquid is stable, while, in the region located above the curve, surface instability develops.

A typical dependence of the wavelength of the surface instability of the cylindrical layer λ, made dimensionless with respect to its perimeter, on the parameter S is shown in Fig. 6a. It must be noted that the wavelength, with a rise in S, decreases from 1.6 to 0.2. In this case, the value of the intensity of the magnetic field varies in the range 0-103 kA/m. In the given case, the maximal recorded value λ = 1.6 is found to be close to the wavelength of the perturbations developing in the absence of an external magnetic field. Consequently, with values of its intensity close to the critical, the external field induces surface instability with a wavelength close to the wavelength characteristic for the decomposition of a cylindrical layer into drops, due to capillary forces. In the region S < S*, λ tends toward infinity, which points to stability of the cylindrical layer. It is interesting that, in the region S > 200, which corresponds to an intensity of the magnetic field of around 100 kA/m, the wavelength varies only weakly. With the above value of the intensity of the external magnetic field, the magnetization of the liquid practically attains saturation. In the investigated range of magnetic $Bo_m$ numbers, the wavelength of the surface instability does not depend on the value of $Bo_m$.

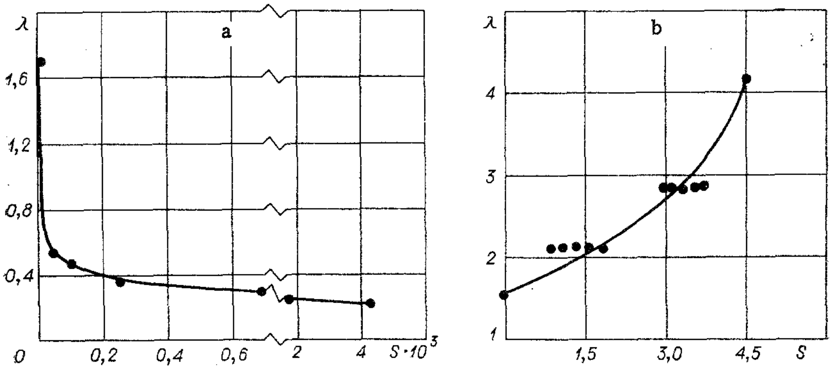


Fig. 6

The stability of a cylindrical layer of a magnetizing liquid in a homogeneous magnetic field tangential to the surface of the layer was investigated experimentally in the following way. Through the conductor there was passed a direct electrical current, whose value made it possible to obtain a value of the magnetic Bond number $Bo_m > 1$. With different values of the intensity of the external magnetic field, the current passing through the conductor was switched off, which made it possible to obtain a value of $Bo_m = 0$. It was established that, with an increase in the intensity of the external magnetic field, the number of drops, into which the layer decomposes, decreases and, at some value of H, the layer remains stable, even with $Bo_m = 0$. The shift of the perturbations excited toward the side of longer wavelengths is illustrated in Fig. 6b, which shows the dependence of the wavelength of the perturbations λ on the parameter S in a tangential field. As in [13], the experimental points are grouped by series, i.e., λ is constant in a certain range of values of S. We note that, with small values of the intensity of the external magnetic field, the region with a constant wavelength is broader than with higher values of S. Consequently, with an increase in the intensity of the external magnetic field, the transition from a state with a smaller wavelength to a state with a large wavelength takes place with a smaller increment in the value of H. With values of H greater than the critical for a given length of the layer, in our case $H_* = 4$ kA/m, the wavelengths of the unstable perturbations exceed the length of the layer, and the layer is stable with respect to all the perturbations.

The results of the experiments made attest to the fact that a homogeneous magnetic field, tangential to the surface of a magnetizing liquid, exerts a stabilizing effect, while a field directed along a normal to the surface, destabilizes it, starting from some critical values of the intensity of the field, depending on the physical properties of the liquid and the parameters of the layer.

The data of the investigation clearly demonstrates the sufficient simplicity of the control of the free surface of a magnetizing liquid, i.e., the possibility of the modeling and detailed study of the various processes taking place at an interface, both in the presence and in the absence of gravitational forces.

The authors express their thanks to R. I. Soloukhin for his valuable observations with evaluation of the work.

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# THEORY OF AN INDUCTION MHD PROPELLER WITH A FREE FIELD

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UDC 538.4

A considerable number of papers have been published [1-6] on magnetohydrodynamic propellers. However, the successes achieved in recent years in creating and using in technology superconducting magnetic systems [7] are an impetus for further investigations into MHD propellers.

This paper is devoted to an investigation of the energy characteristics of a so-called induction MHD system with a free field [2]. This work is made necessary by the fact that in [2] the energy characteristics of the MHD propellers under consideration were obtained without taking into account the longitudinal boundary effect and are therefore grossly overstated. Subsequently the results in [2], without critical analysis, were reproduced in other publications [3, 6] devoted to MHD propellers.

The investigation carried out in this work showed that taking account of the finiteness of the dimensions of the source of the electromagnetic field leads not only to quantitative changes. At the same time the effectiveness of the installation for a given magnetic field intensity is substantially below the predictions [2], the required magnetic fields for obtaining a given efficiency are considerably higher. In the paper we propose a method for increasing the effectiveness of the induction MHD propeller under consideration as a result of "amplitude modulation"; in this case the energy characteristics of the propeller (of finite dimensions) can be to a certain degree brought nearer to an "ideal" propeller [2].

1. We consider a rigid body of finite dimensions located in boundless conducting liquid with conductivity $\sigma$, density $\rho$, being brought in motion by electromagnetic forces; the source of the fields is located within the body. In the role of the rigid body we consider the simplest model – a flat plate of finite width $2a$ along the x axis, infinitely extending along the z axis, moving in its plane in the direction of the negative x half-axis. The assumption about infinity along the z axis is of no major importance. The results obtained will be true if the long plate being considered is rolled into a "ring" or cylinder with a height of $2a$ and a radius substantially exceeding the wavelength $2\pi/k_1$.

The source of the electromagnetic field in the surrounding liquid is provided by introduction, in the plane of the plate, of surface currents having z-direction and being distributed over the width of the plate:

$$i_z(x_1, t) = \mathrm{Real}\, J_0 \cdot i_0(x_1)\, e^{i(k_1 x_1 - \omega_0 t)}\ (|x_1| \leqslant a) \tag{1.1}$$

(these currents act in the role of an inductor). In (1.1) $J_0$ is the maximum current density, the function $i_0(x_1)$ characterizes the distribution of the current amplitude over the plate width, $|i_0(x_1)| \leqslant 1$. By $x_1$, $y_1$ here and below we denote the coordinates with dimensions; for the corresponding dimensionless quantities we use the symbols x, y without indices. The problem consists of determining the distribution of the fields E, H of total force acting on the plate with the currents (1.1) from the side of the magnetic field of the currents j in the liquid, the required electric power, and also the velocity $u_0$ which is acquired by the plate.

Below it is shown that within fairly wide limits of parameters the assumption about smallness of the parameter of magnetohydrodynamic interaction is valid:

$$N = \frac{\sigma H_0^2 2a}{\rho c^2 u_0} \ll 1. \tag{1.2}$$

The problem now becomes simpler. In particular, the electromagnetic fields in the liquid are determined from the equations

$$\mathrm{rot}\,\mathbf{H} = (4\pi/c)\,\mathbf{j}, \quad \mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \quad \mathrm{div}\,\mathbf{H} = 0,$$
$$\mathbf{j} = \sigma[\mathbf{E} + (1/c)\,\mathbf{v} \times \mathbf{H}], \tag{1.3}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 105-116, May-June, 1980. Original article submitted June 28, 1979.

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